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A critical cosmological constant from millimeter extra dimensions

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Abstract: We consider 'brane universe' scenarios with standard-model fields localized on a 3-brane in 6 spacetime dimensions. We show that if the spacetime is rotationally symmetric about the brane, local quantities in the bulk are insensitive to the couplings on the brane. This potentially allows compactifications where the effective 4-dimensional cosmological constant is independent of the couplings on the 3-brane. We consider several possible singularity-free compactification mechanisms, and find that they do not maintain this property. We also find solutions with naked spacetime singularities, and we speculate that new short-distance physics can become important near the singularities and allow a compactification with the desired properties. The picture that emerges is that standard-model loop contributions to the effective 4-dimensional cosmological constant can be cut off at distances shorter than the compactification scale. At shorter distance scales, renormalization effects due to standard-model fields renormalize the 3-brane tension, which changes a deficit angle in the transverse space without affecting local quantities in the bulk. For a compactification scale of order 10^{-2} mm, this gives a standard-model contribution to the cosmological constant in the range favored by cosmology.

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Contents

1. Introduction

The cosmological constant problem is by far the most severe fine-tuning problem in physics. Despite many interesting proposals (see [1] for a review) the problem still lacks a compelling solution. Recently, there has been an interesting idea in the context of the 'brane universe' scenario that may solve part of the cosmological constant problem, namely the sensitivity of the effective low-energy cosmological constant to standard-model radiative corrections [2, 3]. In this proposal the standard-model fields are assumed to be localized on a 3-brane in 5 spacetime dimensions. It was argued in [2, 3] that special bulk-brane interactions can be chosen so that solutions with unbroken 4-dimensional Poincaré invariance exist independently of the standard model parameters and the 3-brane tension. The solutions found in [2, 3] have a naked spacetime singularity, whose resolution in a more fundamental theory of gravity was argued to play an important role in this scenario. On the other hand, the description of the cancellation mechanism of the cosmological constant from a low-energy field theory perspective is not evident. Also, it is not clear whether these ideas can be extended to eliminate the fine-tuning of bulk interactions [4].

In this paper we describe a mechanism for relaxing the cosmological constant that incorporates some of the features of [2, 3], and that addresses some of these open questions. We assume that the standard-model fields are localized on a 3-brane in 6 spacetime dimensions. We exhibit bulk solutions with the property that no locally-defined quantity away from the 3-brane is sensitive to the value of the 3-brane tension. This arises because the only effect of the 3-brane tension on the geometry is to induce a conical singularity in the transverse space with a deficit angle proportional to the brane tension. These solutions have flat induced metric on the 3-brane independent of the couplings on the 3-brane, and are therefore a natural context for the ideas of [2, 3].

In order to obtain 4-dimensional gravity at long distances, the extra dimensions must be compactified. We first consider several mechanisms for non-singular compactifications that can be reliably analyzed in the context of 6-dimensional gravity. We find that none of these mechanisms preserves the property that the 4-dimensional cosmological constant is independent of the couplings on the 3-brane. We give general reasons for this.

The solutions that we find generally have naked spacetime singularities at finite distance from the 3-brane. These allow new compactification mechanisms that are sensitive to physics at the 6-dimensional Planck scale. We speculate that these may allow compactifications with the desired properties. We argue that such a compactification cannot be parameterized purely by higher-dimension operators, suggesting that new Planck-scale degrees of freedom play an important role.

We then speculate on the behavior of gravity at long distances, assuming that a compactification mechanism with the desired properties can be found. If the compactification scale is ℓ, we argue that standard model loop contributions to the effective 4–dimensional cosmological constant are cut off at distances shorter than ℓ. The standard-model contribution to the cosmological constant is then of order $\ell^{-4}/16\pi^2$, which can be small if ℓ is large. This model therefore realizes the ideas of [5]. Recent cosmological observations suggest that vacuum energy may be a significant fraction of the present density of the universe, as suggested by recent observations of type IA supernovae [6]. The standard-model contribution to the cosmological constant is of this order for $\ell \sim 10^{-2}$ mm. As is now well-known, having 2 extra dimensions of this size do not grossly conflict with observation or cosmology [7]. More refined estimates of cosmological bounds give limits as strong as $\ell \lesssim 10^{-5}$ mm [8]. We will not attempt to resolve this discrepancy in the present paper, but we note that the solutions we find have a nontrivial 'warp factor' in the bulk that can weaken the bounds. It is also interesting that extra dimensions of this size can be probed in upcoming short-distance gravity experiments and in high-energy collider experiments.

It is important to remember that the scenario we are discussing is not a complete solution to the cosmological constant problem. The reason is that interactions involving bulk fields and their couplings to other branes must be adjusted to special values to obtain a solution with unbroken 4-dimensional Poincaré invariance. The reason that the mechanism considered here is progress is that this tuning does not involve any coupling involving the 3-brane. This means that we can hope that the required parameter relations be made natural by unbroken symmetries in the bulk, for example supersymmetry.

It is interesting that these ideas provide a completely independent motivation for considering 2 large extra dimensions in the millimeter range. The original motivation came from the gauge hierarchy problem [9]. Large extra dimensions can solve the gauge hierarchy problem, and for 2 extra dimensions the size of the extra dimensions must be of order 1 mm. The fact that logarithmic potentials are natural in 2 extra dimensions gives a natural mechanism for stabilizing large extra dimensions [10]. Finally, precisely in 2 extra dimensions there is a mechanism that may explain the apparent unification of standard-model couplings near the Planck scale [11].

2. The mechanism

To describe the mechanism described above in more detail, consider a 6-dimensional metric of the form

$$ds^2 = \omega^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + \rho^2(r)d\theta^2\,, \tag{2.1}$$

where x^μ ($\mu = 0, \ldots, 3$) parameterize the space parallel to the brane, $\eta_{\mu\nu}$ is the 4-dimensional Minkowski metric, and r, θ parameterize the transverse space. This is the most general metric with unbroken 4-dimensional Poincaré invariance and a translation symmetry ($\theta \mapsto \theta+$constant) in the transverse space. We assume that the variable θ is periodic with period 2π, so the translation symmetry in θ is interpreted as rotation symmetry. If $\rho(r) \propto r$ as $r \to 0$, then $r = 0$ corresponds to a single point in the transverse space, like the origin of polar coordinates. A 3-brane with extent along the x^μ directions at $r = 0$ induces a conical singularity with deficit angle

$$\Delta\theta = 2\pi\left[\rho'(0) - 1\right] = \frac{T_4}{M_6^4}\,, \tag{2.2}$$

where T_4 is the 3-brane tension and M_6 is the 6-dimensional Planck scale.

Away from the brane and away from other singularities, no local quantity is sensitive to the 3-brane tension. A simple way to see this is to note that for any value of the brane tension we can rescale θ so that $\rho'(0) = 1$. In these coordinates, the deficit angle is parameterized by the range of θ, which cannot be determined by measuring local quantities–as long as there is no other deficit angle singularities away from $r = 0$. The reason for the caveat can be understood by representing the solution with a deficit angle by a smooth metric of the form eq. (2.1) with a 'wedge' cut out starting from $r = 0$. Specifically, we consider two radial geodesics in the transverse space emerging from the 3-brane with relative angle $\Delta\theta$, and identify points on the geodesics that are equidistant from the 3-brane. Away from the brane the position of these geodesics is not measurable, since their absolute angular position is a coordinate artifact. However, if the geodesics cross, the space is compactified with spherical topology, and at the second crossing there is another conical singularity with a deficit angle proportional to the deficit angle at the original 3-brane. This must be interpreted as due to a second 3-brane, whose tension is proportional to the tension of the original 3-brane. If such singularities are not present, then there are solutions for any value of the 3-brane tension.

These solutions must be compactified. From the discussion above, we see that we do not want to compactify the solution with a second 3-brane, since that necessarily involves tuning the two 3-brane tensions against each other. One promising possibility is to end the space at a Z_2 orbifold plane at constant r surrounding the 3-brane at $r = 0$. Another possibility is that the space is not compactified, but that the warp factor $\omega(r)$ degreases sufficiently rapidly at large r that gravity is effectively localized at the 3-brane.

To understand the limitations of these ideas, it is useful to compute the effective 4-dimensional cosmological constant assuming that a compactified solution has been obtained. This can be obtained by integrating the 6-dimensional action evaluated at the classical solution over the transverse space. This gives[1]

$$\Lambda_{4,\text{eff}} = T_4 - \int dr d\theta\, \omega^4(r)\rho(r) \left[\frac{M_6^4}{2} R - \Lambda_6 + \cdots \right], \tag{2.3}$$

where R is the bulk Ricci scalar, Λ_6 is the bulk cosmological constant, and the ellipses denote the contribution from possible additional bulk fields or other branes. The Ricci scalar evaluated at the solution can be written

$$R = R_{\text{sing}} + R_{\text{bulk}}\,, \tag{2.4}$$

where R_{sing} is proportional to $T_4\delta(r)$, and R_{bulk} contains the contributions from the bulk fields and other branes. Precisely for the case of 2 extra dimensions, the contribution from R_{sing} exactly cancels the contribution from the first term on the right-hand side of eq. (2.3), and we have

$$\Lambda_{4,\text{eff}} = -\theta_{\max} \int dr\, \omega^4(r)\rho(r) \left[\frac{M_6^4}{2} R_{\text{bulk}} - \Lambda_6 + \cdots \right], \tag{2.5}$$

where we have performed the (trivial) θ integral. In order to solve the cosmological constant problem, the integral on the right-hand side of eq. (2.5) must vanish. This condition is completely independent of couplings on the brane, which is another manifestation of the independence of the brane tension discussed above. However, the vanishing of the integral above is an additional condition that in general requires

[1] Our conventions are the same as the book by Wald [13]: the metric is 'mostly plus', and $R > 0$ for de Sitter space.

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fine-tuning [4]. In the present scenario, we can hope that the vanishing of this integral can be enforced by unbroken symmetries in the bulk. For example, supersymmetry can be unbroken in the bulk, and branes away from the standard-model 3-brane may be BPS states. This is natural even if supersymmetry is broken badly by fields localized on the brane, since radiative corrections of brane fields do not renormalize the couplings associated with bulk modes and distant branes.

3. Bulk solutions

We begin by determining the metric away from the branes. The solution is a special case of the one obtained in [14] (see also [15]), but we will derive it from first principles for completeness. We generalize the metric to allow constant curvature on the 3-brane. We parameterize the 6-dimensional metric as

$$ds^2 = \omega^2 \gamma_{\mu\nu} dx^\mu dx^\nu + s(\omega) d\omega^2 + f(\omega) d\xi^2 \,, \tag{3.1}$$

where $\gamma_{\mu\nu}$ is a metric with constant curvature λ (i.e. $R_{\mu\nu}(\gamma) = \lambda \gamma_{\mu\nu}$). The 'warp factor' ω is the radial coordinate in the transverse space, while ξ is an angular variable. The solution is particularly simple in these coordinates, as we will see. (These coordinates are also employed in [14].) The nonvanishing components of the Einstein tensor are

$$\begin{aligned}
G_{\mu\nu} &= \frac{1}{s}\left[\frac{f''}{2f} - \frac{f'^2}{4f^2} - \frac{f's'}{4fs} + \frac{3f'}{2\omega f} - \frac{3s'}{2\omega s} + \frac{3}{\omega^2} - \frac{\lambda s}{\omega^2}\right] g_{\mu\nu} \,, \\
G_{\omega\omega} &= \frac{1}{s}\left[\frac{2f'}{\omega f} + \frac{6}{\omega^2} - \frac{2\lambda s}{\omega^2}\right] g_{\omega\omega} \,, \\
G_{\xi\xi} &= \frac{1}{s}\left[-\frac{2s'}{\omega s} + \frac{6}{\omega^2} - \frac{2\lambda s}{\omega^2}\right] g_{\xi\xi} \,,
\end{aligned} \tag{3.2}$$

where $f' = df/d\omega$, etc.

The bulk cosmological constant and the 4-dimensional curvature λ cancel in the difference of the $\omega\omega$ and $\xi\xi$ equations, which give $fs' + f's = 0$. This immediately implies

$$s(\omega) = \frac{1}{f(\omega)} \,, \tag{3.3}$$

where a possible overall constant can be set to one by rescaling x^μ and ω. Substituting this into the $\xi\xi$ or $\omega\omega$ equations gives

$$\omega f' + 3f = -\frac{\omega^2}{2}\frac{\Lambda_6}{M_6^4} + \lambda \,. \tag{3.4}$$

The general solution is simply

$$f(\omega) = -k\omega^2 + \frac{1}{3}\lambda + c\omega^{-3} \,, \tag{3.5}$$

where

$$k = \frac{\Lambda_6}{10M_6^4} \tag{3.6}$$

and c is an arbitrary integration constant. With this solution for $f(\omega)$ and $s(\omega)$, one finds that the $\mu\nu$ equations are solved; this is a general consequence of the reparameterization invariance in r. Note that ω is dimensionless, k, λ, and c have mass dimension $+2$, and ξ has mass dimension -2.

The relation to the more 'physical' coordinates of eq. (2.1) is easy to work out from

$$dr = \pm \frac{d\omega}{\sqrt{f(\omega)}}, \tag{3.7}$$

where the sign indicates whether the warp factor is increasing or decreasing with increasing r. For $\lambda = 0$, we obtain

$$\omega(r) = \begin{cases} [\cos\alpha(r - r_0)]^{2/5} & \text{for } \Lambda_6 > 0\,, \\ [\cosh\alpha(r - r_0)]^{2/5} & \text{for } \Lambda_6 < 0\,, \end{cases} \tag{3.8}$$

$$\rho(r) = \rho_0 \omega'(r)\,, \tag{3.9}$$

where

$$\alpha \equiv \sqrt{\frac{5|\Lambda_6|}{8M_6^4}}\,. \tag{3.10}$$

We now discuss the physical interpretation of the solutions for $\lambda = 0$. In the coordinates of eq. (3.1), the physical region corresponds to $f(\omega) > 0$. For $c = 0$, we must have $k < 0$, and the only solution is anti de Sitter space. For $c \neq 0$, we can choose $c = k$ by rescaling the coordinates, so that the physical region corresponds to $0 \leq \omega \leq 1$ for $k > 0$, and $\omega \geq 1$ or $\omega < 0$ for for $k < 0$. For both signs of k, f has a simple zero at $\omega = 1$. The geometry there can be exhibited by transforming to the r, θ coordinates:

$$r - r_0 \simeq \frac{2}{\sqrt{5|k|}} \sqrt{|\omega - 1|} \quad \text{for } \omega \simeq 1\,. \tag{3.11}$$

The metric near $\omega = 1$ is then

$$ds^2 \simeq \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + r^2 d\theta^2\,, \qquad \theta \equiv \frac{5|k|}{2}\xi\,, \tag{3.12}$$

where we have chosen the constant ρ_0 in eq. (3.9) to fix the coefficient of $d\theta^2$. If θ is periodic with period $\theta_{\max} = 2\pi$, there is no singularity at $\omega = 1$ ($r = 0$). For any other value of $\theta_{\max}$ there is a deficit angle singularity at $r = 0$ that we interpret as being due to the presence of a 3-brane at $r = 0$ with tension proportional to the deficit angle.[2] The 3-brane equations of motion impose the condition $\omega'(r_0) = 0$. We can see that this is satisfied from eq. (3.8).

[2] We have verified this explicitly by smearing out the 3-brane by replacing it by a 'ring' at $r = a$ and solving the gravitational field equations. We find that the deficit angle is the only effect that survives in the limit $a \to 0$ with the 3-brane tension held fixed.

For $k < 0$, the solution given by eqs (3.3) and (3.5) for $\omega \geq 1$ describes a 3-brane in infinite space of negative cosmological constant. The 'warp factor' ω increases away from the 3-brane, so gravity is not localized.

For $k > 0$, ω decreases away from the 3-brane, and $f(\omega)$ diverges as $\omega \to 0$. This is a true singularity, as can be seen from the curvature invariant

$$R_{MNPQ}R^{MNPQ} \simeq \frac{240k^2}{\omega^{10}} \quad \text{as } \omega \to 0\,, \tag{3.13}$$

where $M, N = 0, \ldots, 5$. The proper distance to the singularity is $\pi/(5\sqrt{k})$, and the time measured by an observer on the brane for a light signal to go from the brane to the singularity and back is

$$\Delta t = 2\int_0^1 d\omega\, \frac{1}{\omega\sqrt{f(\omega)}} \simeq \frac{1.8}{\sqrt{k}}\,. \tag{3.14}$$

This singularity is similar to the one found in [16]. It is an interesting conjecture (following [2, 3]) that in a more fundamental theory of gravity this singularity is smoothed out in a way that preserves the independence of the solution on the 3-brane tension.

For $k < 0$ the region $\omega < 0$ is also physical, although it is not possible to have a 3-brane in this region. (Note that the metric depends only on ω^2, so the sign of ω is not physical). This also has a naked singularity as $\omega \to 0$; in fact, the metric is the same near both singularities.

4. Compactification

We now turn to compactification. We first attempt to construct compactified solutions using mechanisms that we can control in the 6-dimensional gravity effective theory. We will find that these conventional mechanisms fail for very general reasons. We then turn to possible compactifications that depend on the existence of the naked singularities.

4.1 Minimal 4-branes

We first attempt to find a compact solution with a 4-brane surrounding the 3-brane at fixed r.[3] In order to avoid a second deficit angle singularity, we assume that the 4-brane is at a Z_2 orbifold boundary, which effectively ends the space.

It is easy to see that there are no solutions if the 4-brane action contains only a tension term. The reason is that the 4-brane tension gives rise to boundary conditions (in the metric of eq. (2.1))

$$\frac{\Delta\omega'}{\omega} = \frac{\Delta\rho'}{\rho} = -\frac{T_5}{4}\,. \tag{4.1}$$

[3]Solutions similar to the one constructed here are considered in [19].

At a symmetric point, this implies that $\omega'/\omega = \rho'/\rho$ on both sides of the 4-brane. If we use this as initial data for evolution in r, we obtain a solution with $\rho(r) \propto \omega(r)$. This cannot evolve in a finite distance to the solution near the 3-brane, which has $\rho(r) \not\propto \omega(r)$.

4.2 Non-minimal 4-branes

We now consider the possibility that there is a non-minimal stress tensor on the 4-brane of the form

$$t_{\mu\nu} = -T_5 \gamma_{\mu\nu}, \quad t_{\theta\theta} = -T_{5,\theta} \gamma_{\theta\theta}\,, \tag{4.2}$$

where γ is the induced metric on the 4-brane. This is the most general stress tensor compatible with the symmetries we are assuming. The asymmetry $T_{5,\theta} \neq T_5$ allows us to find solutions with a 3-brane surrounded by a 4-brane at finite distance, as we will see below. In the limit $T_{5,\theta} \to T_5$ the proper separation between the 3-brane and the 4-brane becomes infinite, so the mechanism that gives rise to the asymmetry is crucial for the compactification.

To cancel the standard-model contributions to the cosmological constant, it is important that the mechanism that gives rise to the asymmetry does not bring in dependence on the 3-brane tension via the deficit angle.[4] For example, Casimir energy from fields localized on the 4-brane will give rise to a stress tensor of the form eq. (4.2), but $T_{5,\theta} - T_5$ is proportional to the circumference of the 4-brane, which depends on the deficit angle at the 3-brane. Similarly, a scalar field localized on the 4-brane that wraps in the θ direction must come back to its original value around the 4-brane, and hence its stress tensor also depends on the deficit angle.

A more promising mechanism is to localize a 3-form gauge field on the 4-brane and assume that it has nonzero field strength

$$F_{\mu\nu\rho\sigma} = E \epsilon_{\mu\nu\rho\sigma}, \qquad E = \text{constant}\,, \tag{4.3}$$

with all other components vanishing. This is a constant 'electric' field in the x^μ directions, and is not subject to any quantization condition. The sign of the stress tensor from this configuration gives $T_5 - T_{5,\theta} > 0$. We will see that this is the sign that we require to obtain a solution. The solution requires a single fine-tuning involving E, but this fine-tuning does *not* involve the 3-brane tension.

Nonetheless, this is not a satisfactory solution. To understand the reason, consider adiabatically changing the 3-brane tension. This can be thought of as a crude model for a phase transition involving matter on the 3-brane. The metric then has the form eq. (2.1) with an adiabatically evolving deficit angle. As the deficit angle changes in time, the size of the 4-brane changes. This changes the value of E, since

[4] We thank A. Nelson and R. Sundrum for emphasizing this point.

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it is the integrated number of field lines (rather than E) that is conserved in the adiabatic evolution. The 4-form flux mechanism has effectively recast the cosmological constant problem as a problem of fine-tuning initial data.

This phenomenon is actually a general consequence of the conservation of stress-energy. To see this, consider a stress tensor of the form eq. (4.2) in a metric with adiabatically changing deficit angle. This metric can be parameterized by the replacement $\rho(r) \to \rho(r,t)$ in eq. (2.1) (with a fixed range for θ). The equation $\nabla^M T_{M0} = 0$ then implies

$$\partial_t T_5 = (T_{5,\theta} - T_5)\,\frac{\partial_t \rho}{\rho}\,. \tag{4.4}$$

We see that T_5 must change in response to the deficit angle.

In the remainder of this section, we construct the solutions explicitly. The main goal of this is obtain a quantitative estimate of the fine-tuning of the 4-brane tension that is required to obtain a small cosmological constant. We find that there is no improvement relative to a generic theory with new physics at the TeV scale. The reader who is not interested in these details can skip to the next Subsection.

Away from the 4-brane, the solution has the form considered in the previous Section. At the 4-brane, the metric components have discontinuous first derivatives. In terms of the metric eq. (2.1), the relevant terms in the Einstein tensor are those with second derivatives with respect to r. These are

$$\begin{aligned} G_{\mu\nu} &= \left[\frac{3\omega''}{\omega} + \frac{\rho''}{\rho} + \cdots\right] g_{\mu\nu}\,, \\ G_{\theta\theta} &= \left[\frac{4\omega''}{\omega} + \cdots\right] g_{\theta\theta}\,. \end{aligned} \tag{4.5}$$

The 4-brane gives a nonvanishing contribution to the $\mu\nu$ and $\theta\theta$ components of the stress tensor proportional to $\delta(r - r_0)$, where r_0 is the radial position of the 4-brane. This gives the discontinuity conditions

$$\frac{\Delta\omega'}{\omega} = -\frac{T_{5,\theta}}{4}, \qquad \frac{\Delta\rho'}{\rho} = -T_5 + \frac{3T_{5,\theta}}{4}\,, \tag{4.6}$$

where we now use units where $M_6 = 1$. The functions ω and ρ are continuous at the 4-brane.

It is convenient to write these conditions in terms of the 'warp factor' coordinates using

$$\omega' = \epsilon\sqrt{f(\omega)}\,, \qquad \epsilon = \pm 1\,. \tag{4.7}$$

The sign factor ϵ tells us whether the warp factor is increasing or decreasing with increasing r; it can be chosen independently on either side of the brane. We choose the coordinate r to increase monotonically as we pass through the 4-brane from the 3-brane position at $r = 0$.

Because of the Z_2 orbifold projection, we require the solution to be symmetric with respect to reflections about the 4-brane. This means that the bulk solution is described by the same function $f(\omega)$ on both sides of the 4-brane. The ω' discontinuity equation is then

$$\frac{\sqrt{f_0}}{\omega_0}\left(\epsilon_2-\epsilon_1\right)=-\frac{T_{5,\theta}}{4}\,, \qquad f_0\equiv f(\omega_0)\,, \tag{4.8}$$

where ω_0 is the warp factor at the 4-brane.

We first consider a space with negative cosmological constant. The warp factor increases away from the 3-brane, so $\epsilon_1=+1$, $\epsilon_2=-1$. From eq. (4.8) we see that this requires $T_{5,\theta}>0$. eq. (4.8) can then be written as

$$-k+\frac{1}{3}\lambda\omega_0^{-2}+c\omega_0^{-5}=\left(\frac{T_{5,\theta}}{8}\right)^2. \tag{4.9}$$

Using

$$\frac{\rho'}{\rho}=\epsilon\frac{f'(\omega)}{2\sqrt{f(\omega)}}\,, \tag{4.10}$$

the ρ' discontinuity equation can be written

$$-\frac{f'(\omega_0)}{\sqrt{f_0}}=-T_5+\frac{3T_{5,\theta}}{4}\,. \tag{4.11}$$

Combined with eq. (4.9), this can be simplified to give

$$-2k-3c\omega_0^{-5}=\frac{T_{5,\theta}}{8}\left(T_5-\frac{3}{4}T_{5,\theta}\right). \tag{4.12}$$

For $\lambda=0$, we can combine this with eq. (4.9) to obtain

$$k=-\frac{T_{5,\theta}}{40}\left(T_5-\frac{3}{8}T_{5,\theta}\right). \tag{4.13}$$

This is a fine-tuning condition that is required to obtain a solution with vanishing 4-dimensional curvature. For $\lambda\neq 0$, eqs. (4.9) and (4.12) can be solved for ω_0 and λ. A similar fine-tuning in [20] is required to ensure the flatness of a brane with codimension 1. In fact, if we take the limit $c\to 0$, $T_{5,\theta}\to T_5$, we obtain exactly the solution of [20] in one dimension higher.

We now assume that $c\neq 0$, so that we can rescale coordinates and choose $c=k$. We also set $\lambda=0$. The solution then has a deficit angle at $\omega=1$ that we interpret as a 3-brane. The 4-brane position $\omega=\omega_0$ is then determined by eqs. (4.9) and (4.13) to be

$$\omega_0^{-5}=\frac{T_{5,\theta}}{40k}\left(T_{5,\theta}-T_5\right). \tag{4.14}$$

The proper radial distance between the 3-brane and 4-brane is

$$\ell=\frac{1}{\sqrt{-k}}\int_1^{\omega_0}\frac{d\omega}{\sqrt{\omega^2-\omega^{-3}}}=\frac{2}{5\sqrt{|k|}}\ln\left[\sqrt{\omega_0^5}+\sqrt{\omega_0^5-1}\right]. \tag{4.15}$$

As discussed above, in the limit $T_{5,\theta} \to T_5$, we have $\omega_0 \to \infty$ and $\ell \to \infty$. In order to obtain finite ℓ with $\omega_0 > 1$, we need $T_5 > T_{5,\theta}$ (remember $k < 0$). These signs are compatible with the stress tensor from a 3-form gauge field, as discussed previously. The effective 4-dimensional Planck scale is given by

$$M_4^2 = \frac{2M_6^4}{15|k|} \left(\omega_0^3 - 1\right) \theta_{\max}. \tag{4.16}$$

The warp factor has maximum value at the 4-brane, so the solution tends to localize gravity there. The warp factor grows exponentially as a function of $\sqrt{-k}\,\ell$ (see eq. (3.8)), so we can solve the gauge hierarchy problem via the mechanism of [12] for $\ell \sim 10/\sqrt{-k}$. We will not pursue this possibility here.

For $k > 0$, a similar analysis shows that eqs. (4.13) and (4.14) hold in this case as well, but now $T_{5,\theta} < 0$ and $T_5 < T_{5,\theta}$. The fact that the 4-brane has negative tension is not a problem, since it is fixed on the orbifold plane. In these solutions, the warp factor decreases away from the 3-brane, and the 4-brane cuts of the space before the naked singularity at $\omega = 0$.

We now consider the fine-tuning of the cosmological constant in these solutions. If we perturb the 4-brane tensions away from their fine-tuned values, $T_5 \to T_5 + \Delta T_5$, $T_{5,\theta} \to T_{5,\theta} + \Delta T_{5,\theta}$, we obtain

$$\begin{aligned} \lambda &= \frac{\omega_0^2}{8} \left[T_{5,\theta} \Delta T_5 + \left(T_5 - \frac{3}{4} T_{5,\theta} \right) \Delta T_{5,\theta} \right] + \mathcal{O}(\Delta T^2)\,, \\ \Delta\omega_0 &= -\frac{\omega_0^6}{120k} \left[T_{5,\theta} \Delta T_5 + \left(T_5 - \frac{3}{2} T_{5,\theta} \right) \Delta T_{5,\theta} \right] + \mathcal{O}(\Delta T^2)\,. \end{aligned} \tag{4.17}$$

We estimate the 4-dimensional curvature λ for an order-1 perturbation of the 3-brane tension $\Delta T_4 \sim T_4$. This results in a change of the deficit angle $\Delta\theta \sim \Delta T_4 / M_6^4$, which may be small if $M_6 \gg T_4$. (This can be natural due to approximate SUSY on the 3-brane.) Also, ΔT_5 and $\Delta T_{5,\theta}$ are naturally of order $(T_5 - T_{5,\theta})\Delta\theta$, where we allow $T_5 - T_{5,\theta} \ll T_5$. This results in a 4-dimensional curvature

$$\lambda \sim \frac{\omega_0^2 T_5^2 \Delta T_4}{M_6^4}\,. \tag{4.18}$$

Combining this with eqs. (4.13), (4.14), and (4.16), we obtain

$$\lambda \gtrsim \frac{\Delta T_4}{M_4^2}\,. \tag{4.19}$$

Since $\Delta T_4 \gtrsim 1$ TeV, this is no better than a 4-dimensional theory with a TeV scale vacuum energy (e.g. a theory with supersymmetry broken at the TeV scale).

4.3 Warp factor compactification

We now consider the possibility that the extra dimensions are infinite but with a 'warp factor' $\omega(r)$ that decreases sufficiently rapidly as $r \to \infty$ so that gravity is approximately 4-dimensional at long distances [12]. In this scenario, since gravity is not 4-dimensional even at arbitrarily long distances, it is obvious how it evades 'no-go' theorems concerning the tuning of the cosmological constant [1].

The simplest possibility would be to surround the 3-brane by a 4-brane in the extra dimensions, with the spacetime being anti de Sitter on the outside of the 4-brane.[5] However, the anti de Sitter metric has the symmetry $\rho(r) \propto \omega(r)$, which is preserved by the brane matching condition. Therefore, anti de Sitter space cannot match onto the asymmetric solution that is required if we have a 3-brane in the solution.

Another possibility is to have additional fields in the bulk, and look for solutions where the warp factor vanishes at infinity. This can evade the obstruction described above, since an asymmetric solution with $\rho(r) \not\propto \omega(r)$ can evolve to a symmetric solution asymptotically. The simplest possibility is to introduce scalars ϕ_a into the bulk:

$$S_{\text{scalar}} = \int d^6x \sqrt{-g} \left[-\frac{1}{2} g^{MN} \partial_M \phi_a \partial_N \phi_a - V(\phi) \right]. \tag{4.20}$$

The resulting Einstein equations can be simplified by introducing the quantities

$$\Sigma(r) := \frac{1}{\sqrt{-g}} \partial_r \sqrt{-g} = \frac{4\omega'}{\omega} + \frac{\rho'}{\rho}, \qquad \Delta(r) := \frac{\omega'}{\omega} - \frac{\rho'}{\rho}. \tag{4.21}$$

We require $\sqrt{-g}$ to decrease as $r \to \infty$ to have finite volume, so $\Sigma < 0$. Δ is a measure of the asymmetry between the θ and x^μ directions. In terms of Σ and Δ, Einstein's equations are first-order differential equations. Taking the difference of the $\mu\nu$ and $\theta\theta$ components of Einstein's equations gives

$$\frac{\Delta'}{\Delta} = -\Sigma\,. \tag{4.22}$$

Since $\Sigma < 0$, we see that $|\Delta|$ increases monotonically as $r \to \infty$. This means that we cannot hope to obtain a solution that approaches anti de Sitter space (which has $\rho(r) \propto \omega(r)$ and hence $\Delta \equiv 0$) with a decreasing warp factor.

We may still hope to find compactified solutions with a decreasing warp factor that are not anti de Sitter at infinity. Using eq. (4.22) to simplify the difference of the $\theta\theta$ and rr Einstein equations gives

$$\Sigma' = -\Delta^2 - \frac{5}{4} \phi'_a \phi'_a\,. \tag{4.23}$$

[5] In principle the solution need only approach anti de Sitter at large distances, but the only solution with the required symmetries is exactly anti de Sitter.

Because the right-hand side is negative-definite, Σ decreases (becomes more negative) monotonically. It can be shown that all solutions with a decreasing warp factor have a singularity at finite r by using the fact that the solutions are bounded by the solutions with $\phi_a' \equiv 0$. In detail, let $\Sigma(r)$ and $\Delta(r)$ be solutions with some initial conditions at $r = r_0$, and let $\Sigma_0(r)$ and $\Delta_0(r)$ be the solutions of eqs. (4.22) and (4.23) with the same initial conditions, but setting $\phi_a' \equiv 0$. We then have $\Sigma(r) < \Sigma_0(r)$ and hence $|\Delta(r)| > |\Delta_0(r)|$ for $r > r_0$. It is not hard to see that $|\Delta_0(r)| \to \infty$ at finite r, so $\Delta(r)$ must also have a singularity at finite r.

4.4 Singular compactification?

A striking feature of the solutions constructed in the previous Section is the presence of naked singularities. Near a naked singularity the curvature is blowing up, so the physics of the singularity is sensitive to the details of physics above the Planck scale. It is natural to speculate (following [2, 3]) that this new physics ends the space at the singularity. In this scenario, the long-distance behavior of gravity is directly controlled by the short-distance behavior at the singularity, and we cannot rigorously address the physics of the singularity using the low-energy effective theory. We will therefore confine ourselves to some simple observations.

First, we note that the curvature distinguishes between the x^μ and θ directions. (For example, $R_{r\mu r\mu} \neq R_{r\theta r\theta}$ for $\mu = 0, \ldots, 3$.) This is potentially important for compactification because the failure of the compactification mechanisms above can be traced to the non-existence of terms in the equations of motion that distinguish between the x^μ and θ directions independently of the 3-brane tension. Near the naked singularity, these higher-derivative effects may be important and allow more general boundary conditions than the ones we considered. The resulting theory would be a Z_2 orbifold with Planck-scale curvature near the orbifold boundary.

We can attempt to get some insight into this scenario by adding higher-derivative terms to the action. We cannot hope to obtain consistent solutions, but we can see that the hoped-for effects do not occur if we treat the higher-derivative terms perturbatively. At each order in perturbation theory, we have a system of second-order equations that involves the lower-order solutions. At each order, the boundary conditions on the metric at the orbifold boundary is $\omega'/\omega = \rho'/\rho = 0$, which gives rise to a symmetric solution ($\rho \propto \omega$). For higher-derivative terms localized on the orbifold boundary, the same argument holds if we regulate the brane (e.g. by a scalar domain wall).

If we attempt to include the higher-derivative terms non-perturbatively, the initial data at the orbifold boundary involves higher derivatives of the metric. Taking asymmetric initial conditions, one can presumably find solutions of the kind we seek. However, it is difficult to interpret such solutions physically. The additional initial conditions can be thought of as additional Planck-scale degrees of freedom. However, theories with higher-derivative terms are generally classically unstable, correspond-

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ing to the fact that the extra degrees of freedom are generally ghosts. At best, this line of reasoning may be viewed as a weak hint that new Planck-scale degrees of freedom localized at the orbifold boundary may allow compactification.

Recently, there has been an interesting proposal to make sense out of naked singularities such as this by imposing boundary conditions at the singularity [16]. While we regard this proposal as very interesting, we note that it appears to be difficult to give a generally covariant formulation of the boundary conditions for the metric (see however [17]). Another interesting approach is to look for solutions that 'regulate' the singularity by hiding it behind an event horizon [18]. These approaches are definitely worthy of further exploration.

We conclude that there are potential difficulties with the idea of compactification near the naked singularities, but the idea cannot be ruled out. In light of this, we believe that the scenario described here is worth further investigation.

5. Effective field theory

We now turn to the effective field theory analysis of the scenario described above, assuming that compactification is possible. Without a specific compactification mechanism, we cannot address the details of the Kaluza-Klein (KK) spectrum or the 4-dimensional effective field theory, but we can analyze some simple aspects of the scenario that depend on the behavior near the 3-brane.

We first consider the light degrees of freedom. The KK spectrum contains the 4-dimensional graviton and a massless gauge field corresponding to the unbroken rotational invariance in the transverse space. These can be parameterized by the metric

$$ds^2 = \omega^2(r) g_{\mu\nu}(x) dx^\mu dx^\nu + dr^2 + \rho^2(r) \left[d\theta + A_\mu(x) dx^\mu \right]^2 , \tag{5.1}$$

where $\omega(r)$ and $\rho(r)$ are as in our solution. The KK gauge field couples to momentum in the θ direction, so the standard-model fields are not 'charged' under the gauge group. In addition, the couplings of the KK gauge field is suppressed by large volume factors and the wavefunction factor $\rho(r)$ above. The bounds on the couplings of such a vector are therefore much weaker than the corresponding bounds on KK gravitons, which is safe.

Another important question we can address is whether higher-order terms on the brane upset the non-dependence of the 4-dimensional cosmological constant on the 3-brane tension. For example, we can write

$$\Delta S_{\rm brane} \sim \int d^4x \sqrt{-\gamma}\, M_6^2 \left[g^{MN} R_{MN} + \gamma^{\mu\nu} R_{\mu\nu} + \cdots \right] . \tag{5.2}$$

Such terms can be generated by standard-model loops with external gravitational lines; in fact, *all* such loop effects proportional to positive powers of M_6 (the cutoff) correspond to ultraviolet divergences, and therefore correspond to operators localized on the brane. Since all such effects are equivalent to tree-level terms, this reduces the question to a classical analysis.

Making sense of the equations of motion that follow from eq. (5.2) requires regulating the 3-brane, presumably taking care to preserve general covariance. However, a very general argument shows that the independence of the 4-dimensional cosmological constant of the 3-brane tension is robust against the addition of such effects. Let us momentarily adopt the contrary hypothesis, namely that the absence (or fine-tuning) of the couplings in eq. (5.2) are required to obtain a solution with unbroken 4-dimensional Poincaré invariance. We should then ask what the solutions are. If we violate the fine-tuning of the couplings away from the 3-brane by a small amount, it is clear that the effect in the 4-dimensional effective field theory is a nonzero cosmological constant. The vacuum solutions in the 4-dimensional field theory are then de Sitter or anti de Sitter space. The 6-dimensional metric that corresponds to this solution should therefore have the symmetries of 4-dimensional de Sitter or anti de Sitter space; this metric was given in eqs (3.1), (3.3), and (3.5), where the parameter λ is the 4-dimensional curvature. This metric admits deficit angle singularities at simple zeros of $f(\omega)$. Since these are the *only* solutions with the required symmetries, we conclude that all the effects of the 3-brane couplings can be absorbed into the deficit angle and the 4-dimensional curvature. (This is analogous to a 'no-hair' theorem for codimension 2 branes.)

We must still address the possibility that the 4-dimensional curvature is sensitive to the terms in eq. (5.2). When these terms are properly regulated, their effect on physics below the scale M_6 can be written as boundary conditions on the gravitational fields involving higher derivatives. If the terms in eq. (5.2) can be treated as non-singular perturbations, these new boundary conditions will still allow solutions for any value of the 4-dimensional curvature, including flat space. Under our assumptions, it is the matching condition at the *other* boundary of the transverse space that picks out the value of λ. We conclude that the presence of the terms eq. (5.2) does not invalidate our picture.

This argument eliminates contributions to the effective 4-dimensional cosmological constant proportional to positive powers of M_6. However, we expect loop matching corrections from loops of standard-model fields of order

$$\Lambda_{4,\mathrm{eff}} \sim \frac{\ell^{-4}}{16\pi^2}\,. \tag{5.3}$$

The scale of these corrections is set by ℓ because the size of the dominant loops in position space is of order ℓ. There are no contributions involving positive powers of M_6 because these would have to correspond to a local counterterm in the 6-dimensional theory that gives an ℓ-dependent contribution to $\Lambda_{4,\mathrm{eff}}$ at tree level.[6]

[6]In a Kaluza-Klein description, the 6-dimensional bulk fields are rewritten as an infinite tower of

We can ask whether it is possible that $\Lambda_{4,\mathrm{eff}}$ could be *smaller* than the estimate eq. (5.3). We believe that this is impossible, simply because of the naturalness of the effective theory at distances larger than ℓ. There are no massless scalars in this effective theory, and hence no light degrees of freedom that can adjust the cosmological constant to zero.

We can get some insight into the mechanism for the cancellation of the standard model contribution to the cosmological constant by considering the dynamics of a slowly rolling scalar field localized on the 3-brane. The potential of the scalar acts as an effective 3-brane tension that varies with time. If the rate of change of this tension is sufficiently slow, the bulk gravitation fields will respond adiabatically. The solution will therefore have a deficit angle that tracks the instantaneous value of the scalar potential. From the point of view of the 4-dimensional effective theory, this corresponds to a mixing between the scalar and the gravity KK mode that corresponds to the deficit angle. This light mode adjusts itself to cancel the cosmological constant. We expect this mode to have a mass of order $1\,\mathrm{mm}^{-1}$, and below this scale the cancellation mechanism is no longer effective; this leads to the estimate eq. (5.3). However, we do not understand the generation of a nonzero cosmological constant from a 6-dimensional perspective.

The arguments above address only the contribution of standard-model loops to the effective cosmological constant. We emphasize again that even if one accepts the existence of a compactification with the properties described in the previous Section, we do not have a complete solution to the cosmological constant problem. In particular, we have not addressed the question of bulk gravity loops, and we have seen that fine-tuning of bulk interactions is necessary to obtain a solution with unbroken 4-dimensional Poincaré invariance. The important point is that the quantities that must be fine-tuned do not involve the 3-brane couplings. We can therefore hope that unbroken bulk symmetries such as supersymmetry can make these parameter choices natural. We therefore believe that it is plausible that the leading contribution to the cosmological constant is of order eq. (5.3).

6. Conclusions

We have described a natural mechanism for canceling the standard-model contribution to the cosmological constant. It relies only on the properties of branes with 2 transverse dimensions. The mechanism requires the compactification scale to be

4-dimensional fields. One might worry that a single KK state will give a contribution of order m_{KK}^4 to the effective 4-dimensional cosmological constant, which will be much larger than eq. (5.3) for large m_{KK}. However, the different KK states are nothing more than different eigenstates of momentum in the compact directions, and so such contributions correspond to contributions proportional to the cutoff M_6 from loops with high momentum. The argument above shows that these cannot occur. The underlying reason is 6-dimensional locality, which is not manifest in a KK description.

in the millimeter range, and suggests a nonzero cosmological constant in the range favored by cosmology. Previous work has shown that the presence of 2 large extra dimensions can also explain the gauge hierarchy problem and the unification of gauge couplings; this confluence of ideas is nothing if not suggestive. Most importantly, these ideas are testable by terrestrial experiments and cosmological observations.

While we have not definitely established all aspects of the mechanism we proposed, we hope that some of these ideas will prove fruitful in the search for the ultimate solution of the cosmological constant problem.

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